Rearranging equations

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1   Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \dfrac{v - u}{a}$	**1** Get the terms containing t on one side and everything else on the other side. **2** Divide throughout by a.

Example 2   Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \dfrac{r}{2 - \pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3   Make t the subject of the formula $\dfrac{t + r}{5} = \dfrac{3t}{2}$.

$\dfrac{t + r}{5} = \dfrac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \dfrac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Example 4   Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

$r = \dfrac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t + 5$ $rt - 3t = 5 + r$ $t(r-3) = 5 + r$ $t = \dfrac{5+r}{r-3}$	**1** Remove the fraction first by multiplying throughout by $t-1$. **2** Expand the brackets. **3** Get the terms containing t on one side and everything else on the other side. **4** Factorise the LHS as t is a common factor. **5** Divide throughout by $r-3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \dfrac{S}{T}$ [T]

4 $p = \dfrac{q-r}{t}$ [t]

5 $u = at - \dfrac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ [y]

8 $x = \dfrac{2a-1}{3-a}$ [a]

9 $x = \dfrac{b-c}{d}$ [d]

10 $h = \dfrac{7g-9}{2+g}$ [g]

11 $e(9+x) = 2e + 1$ [e]

12 $y = \dfrac{2x+3}{4-x}$ [x]

13   Make r the subject of the following formulae.

    a $A = \pi r^2$ **b** $V = \dfrac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \dfrac{2}{3}\pi r^2 h$

14   Make x the subject of the following formulae.

    a $\dfrac{xy}{z} = \dfrac{ab}{cd}$ **b** $\dfrac{4\pi cx}{d} = \dfrac{3z}{py^2}$

15   Make $\sin B$ the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

16   Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17   Make x the subject of the following equations.

    a $\dfrac{p}{q}(sx+t) = x - 1$ **b** $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

Answers

1 $\quad d = \dfrac{C}{\pi}$

2 $\quad w = \dfrac{P - 2l}{2}$

3 $\quad T = \dfrac{S}{D}$

4 $\quad t = \dfrac{q - r}{p}$

5 $\quad t = \dfrac{2u}{2a - 1}$

6 $\quad x = \dfrac{V}{a + 4}$

7 $\quad y = 2 + 3x$

8 $\quad a = \dfrac{3x + 1}{x + 2}$

9 $\quad d = \dfrac{b - c}{x}$

10 $\quad g = \dfrac{2h + 9}{7 - h}$

11 $\quad e = \dfrac{1}{x + 7}$

12 $\quad x = \dfrac{4y - 3}{2 + y}$

13 a $\quad r = \sqrt{\dfrac{A}{\pi}}$

b $\quad r = \sqrt[3]{\dfrac{3V}{4\pi}}$

c $\quad r = \dfrac{P}{\pi + 2}$

d $\quad r = \sqrt{\dfrac{3V}{2\pi h}}$

14 a $\quad x = \dfrac{abz}{cdy}$

b $\quad x = \dfrac{3dz}{4\pi c p y^2}$

15 $\quad \sin B = \dfrac{b \sin A}{a}$

16 $\quad \cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

17 a $\quad x = \dfrac{q + pt}{q - ps}$

b $\quad x = \dfrac{3py + 2pqy}{3p - apq} = \dfrac{y(3 + 2q)}{3 - aq}$